Control of a Biped Robot in the Double-Support Phase
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Abstract—The research treats the development and implementation of an advanced control system for a seven-link, 12 degree-of-freedom, biped robot in the double-support phase. A constrained dynamic model of the biped robot is formulated and a reduced order model for the double-support phase is derived. In this model the dependent variables are related to the independent variables through the kinematic Jacobian. Control strategies based on feedforward compensation and linear state feedback are derived for tracking specified joint trajectories. The approach is demonstrated using results of a sway motion obtained from a prototype biped robot that has been constructed.

I. INTRODUCTION

BiPed ROBOT locomotion continues to receive attention from the research community [1]–[3]. Unlike industrial robots, a biped robot is not fixed to the floor, and hence it is difficult to maintain balance. Control systems for biped machines, therefore, are particularly important. In this respect, several interesting research results have been reported. Miura and Shimoyama [4] used feedforward compensation to follow specified trajectories, with feedback providing correction for disturbances. Furusho and Masubuchi [5] presented a reduced second-order model for a biped robot using local position-derivative (PD) feedback at each joint and they showed that two poles of the reduced model correspond to poles of an inverted pendulum. It was shown that there is not much difference concerning the variation of angular momentum and the center of gravity for the original model compared with the reduced order model. Other researchers proposed linear state feedback control laws obtained from an optimal regulator formulation to follow specified trajectories [6], and to minimize the error between current and end-of-stride joint angles without specifying intermediate joint trajectories [7]. One theoretical study used nonlinear feedback to decouple and linearize the biped robot dynamics [8].

Most previous studies involving biped control, however, have concentrated on the single-support phase because it was the predominant portion of the locomotion period. As a result, most of the systems only can walk from a preset position.

In biped locomotion the double-support and single-support phases alternate. The biped robot usually starts and stops motion at the double-support configuration. The analysis of biped locomotion in the double-support phase is very important for improving the smoothness of the biped locomotion system, especially when the power of the actuator acting on the ankle is weak and control becomes important for moving the center of gravity and raising the heel. Hemami and Wyman [9] derived an approach simultaneously applicable to the constrained system and to the unconstrained system. A key element in their model is the derivation of Lagrange multipliers as functions of the state and the input of the system. In their formulation the dimension of the state of the constrained system is the same as that for the unconstrained dynamic system, but the motion of the system is limited to submanifolds of the state space. Since their analysis was based on a geometric approach, the application of their technique to a biped robot is complex.

In this paper, a reduced dynamic model, which involves only the selected independent variables for the double-support phase is formulated. With the double-support constraint, the joint variables are partitioned into independent and dependent variables that are related through a Jacobian matrix. Control strategies based on feedforward compensation and linear state feedback are proposed to track the desired trajectory and stabilize perturbations of the joint variables. Finally, experimental results from a sway motion demonstrate the use of the approach. The advantage of the proposed approach is that the control is performed in a smaller dimensional space and the constraint relation is also merged in the control loop by the dependent variables.

The analysis and experimental results in this paper are based on a prototype biped robot (Fig. 1) which has been designed and built. The total height of the biped machine is 92 cm and the total weight is about 45 kg. Each leg has three segments—upper leg, lower leg, and foot. Twelve actuators (dc motors) provide the capability to change the position and orientation of each leg. Fig. 2 shows a schematic representation of the 12 actuators and their operating limits. Three revolute actuators are associated with the hip joint, one revolute actuator with the knee joint, and two revolute actuators with the ankle joint. The three rotational axes at the hip joint are perpendicular to each other and have a common point of intersection. Similarly, the two rotational axes at the ankle joint are perpendicular to each other and intersect at the same point. The foot is 20 cm × 10 cm. There are eight loadcell sensors located under the foot to detect the
vertical ground reaction force. Physical parameters of the biped machine are listed in Table I. The relative angles between two adjacent links are monitored by optical encoders and their speeds are measured by tachometers attached to the motors. The motors are driven by servo-amplifiers for velocity control. The resolution of each encoder is 0.09 degree per pulse. A harmonic drive between each motor and its joint provides 100:1 reduction.

II. CONSTRAINED DYNAMIC MODEL OF THE BIPED SYSTEM

The constraints for the biped robot in the double-support phase are assumed to be holonomic, arising from geometrical constraints on the joint coordinates. For a biped robot with \( n \) joint coordinates \( z \) and \( n \) input torques \( u \) these constraints can be represented by

\[
e(z) = 0
\]

where \( e \) is an \( m \)-vector and \( m \) is the dimension of the constraint; \( m = 3 \) for a two-dimensional (2-D) biped model and \( m = 6 \) for a three-dimensional (3-D) model. By defining the augmented Lagrangian

\[
L = T - P + \lambda^T e
\]

where \( T \) is the total kinematic energy, \( P \) is the potential energy, and \( \lambda \) is an \( m \)-vector of Lagrangian multipliers, the equations of motion can be written

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{z}} - \frac{\partial L}{\partial z} = u. \tag{3}
\]

Thus, the dynamic equations of the biped robot in the double-support phase can be represented by

\[
H(z) \ddot{z} + g(z, \dot{z}) = J^T \lambda + u \tag{4}
\]

where \( H \) is an \( n \times n \) symmetric positive definite inertia matrix that satisfies

\[
T = \frac{1}{2} \ddot{z}^T H \ddot{z} \tag{5}
\]

and

\[
g = \frac{d}{dt} H \ddot{z} - \frac{\partial T}{\partial z} + \frac{\partial P}{\partial z} \tag{6}
\]

is an \( n \)-vector, and

\[
J = \frac{\partial e}{\partial z} \tag{7}
\]

is an \( m \times n \) Jacobian matrix.

When the biped is in the double-support phase the number of degrees of freedom becomes \((n - m)\). We assume that the constraints can be written as

\[
e(z) = c_1(z_1) - c_2(z_2) = 0 \tag{8}
\]

where \( z = [z_1^T, z_2^T]^T \), and \( z_1 \) are \((n - m)\) independent coordinates and \( z_2 \) are \( m \) dependent coordinates. The physical interpretation of this assumption is that the position and orientation of the biped body can be calculated either starting from the left foot or the right foot, and each involves either of the joint coordinates \( z_1 \) or \( z_2 \). By differentiating (8), we obtain

\[
J(z) \ddot{z} = J_1(z_1) \dot{z}_1 + J_2(z_2) \dot{z}_2 = 0 \tag{9}
\]
TABLE I-A

PHYSICAL DATA OF THE BIPED MACHINE: MASS (IN KG) AND GEOMETRIC DATA (IN METERS) OF SEGMENTS

<table>
<thead>
<tr>
<th>Segment</th>
<th>Joint Location</th>
<th>Relative to Center of Mass</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body</td>
<td>(0, 0)</td>
<td></td>
<td>15.00</td>
</tr>
<tr>
<td>Upper leg</td>
<td>(0, w0, -h0)/(0, -w0, -h0)</td>
<td>Body (0, 0, -r1)</td>
<td>8.50</td>
</tr>
<tr>
<td>Lower leg</td>
<td>(0, 0, -d1)</td>
<td>Upper Leg (0, 0, -r2)</td>
<td>5.50</td>
</tr>
<tr>
<td>Foot</td>
<td>(0, 0, -d1)</td>
<td>Lower Leg (p1, 0, -r3)</td>
<td>7.25</td>
</tr>
</tbody>
</table>

\[ w_0 = 0.11, \ h_0 = 0.07, \ d_1 = 0.35, \ d_2 = 0.28, \ d_3 = 0.12, \ f = 0.14, \ b = 0.06, \]
\[ d = 0.03, \ \omega_f = 0.05, \ r_1 = 0.175, \ r_2 = 0.14, \ r_3 = 0.087, \ \text{and} \ p_1 = 0.000. \]

which solves the second row for \( \lambda \), and substituting it into the first row of (12), we obtain

\[ \dot{H}(z) \dot{x}_1 + \bar{g}(x, \dot{x}_1) = \left( W_1 - J_1^T (J_2^T)^{-1} W_2 \right) u \]

where

\[ \dot{H}(z) = H_{11}(z) - J_1^T (J_2^T)^{-1} H_{21}(z) H_{22}(z) J_2^T J_1 \]
\[ + J_1^T (J_2^T)^{-1} H_{22}(z) J_2^T J_1 \]
\[ \bar{g}(x, \dot{x}) = g_1(x, \dot{x}) - J_1^T (J_2^T)^{-1} g_2(x, \dot{x}) \]
\[ W_1 = \left[ I_{(n-m)\times(n-m)} \right] \text{and} \]
\[ W_2 = \left[ 0_{m\times(n-m)}, I_{m\times m} \right]. \]

By (13) there is actuator redundancy because the dimension of the input is larger than that of the controlled output. From (13) the feedforward compensator to achieve a desired trajectory \((\hat{z}, \dot{z}, \ddot{z})\) can be computed by

\[ u = \left( W_1 - (J_2^{-1} J_1)^T W_2 \right)^+ \left( \dot{H}(z) \hat{x}_1 + \bar{g}(z, \hat{x}) \right) \]

where the superscript + denotes the matrix pseudo-inverse.

III. LOCAL LINEAR STATE FEEDBACK

In this section, a control law is derived for \( z_1 \) and \( z_2 \). For this purpose we use linear state feedback defined by perturbation variables:

\[ \tilde{z} = z - \bar{z} \]

and

\[ \ddot{u} = u - \dot{u}. \]

To simplify the control computation we propose that \( \dot{u}_1 \) controls \( \tilde{z}_1 \) to track the reference inputs \( z_1 \), and \( \dot{u}_2 \) controls \( \tilde{z}_2 \) to maintain the constraint relation (8). Let

\[ f(\tilde{z}_1, \dot{z}_1, \ddot{z}_1, z_1, z_2) = \ddot{H}(z) \dot{z}_1 + \bar{g}(z, \dot{z}). \]

By linearization at the nominal trajectory \((z, \dot{z}, \ddot{z})\) and employing \( \hat{u}_1 \) as the control, we obtain the following perturbation model

\[ \ddot{M} \ddot{z}_1 + \dot{D} \ddot{z}_1 + \ddot{H} \dot{z}_1 = u_1 \]
where

\[ M = \frac{\partial f}{\partial z_1}, \]
\[ \dot{D} = \frac{\partial f}{\partial z_2} + \frac{\partial f}{\partial z_2} \frac{\partial z_2}{\partial z_1}, \]
\[ \dot{H} = \frac{\partial f}{\partial z_1} + \frac{\partial f}{\partial z_2} \frac{\partial z_2}{\partial z_1}. \]

Defining the state vector

\[ x = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix}. \]

Equation (16) can be represented as a linear system

\[ \dot{x} = Ax + Bu, \]

where

\[ A = \begin{bmatrix} -I & -J \\ \frac{M^{-1}}{M^{-1}} H & -\frac{M^{-1}}{M^{-1}} D \end{bmatrix}, \]
\[ B = \begin{bmatrix} -a-I \\ 0 \end{bmatrix}. \]

The stabilizing input \( u_1 \) can be determined to minimize the quadratic performance functional

\[ J = \int_0^\infty \left( x^T Q x + u_1^T R u_1 \right) \, dt \]

where \( Q \succeq 0, R > 0 \). The optimal linear feedback control becomes

\[ u_1 = K_1 x \]

where

\[ K_1 = -R^{-1} B^T P \]

and \( P \) is the solution of the steady-state Riccati equation. From (11) the relation between \( \dot{z}_2 \) and \( z_1 \) is

\[ \dot{z}_2 = -J_2^{-1} J_1 \dot{z}_1. \]

By differentiating (20) and using \( \dot{u}_2 \) as the control input, we obtain

\[ \ddot{z}_2 = \frac{d}{dt} \left( -J_2^{-1} J_1 \right) \dot{z}_1 - \left( J_2^{-1} J_1 \right) \ddot{z}_1 + \dot{u}_2. \]

The resulting control \( u_2 \) to null the perturbation \( \dot{z}_2 \) is

\[ \dot{u}_2 = \begin{bmatrix} \frac{d}{dt} \left( J_2^{-1} J_1 \right) \dot{z}_1 + \left( J_2^{-1} J_1 \right) \ddot{z}_1 \end{bmatrix} = K_2 x. \]

Then the total control becomes

\[ u = \left( W_1 - \left( J_2^{-1} J_1 \right) W_2 \right)^T \left( H(z) \right) \dot{x} + \dot{g}(x, \dot{x}) \]
\[ + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} x. \]

The linear state control problem is solved here under the assumption that the motion is the vicinity of an operating point so that linearization is valid. It is also assumed during the desired motion that the constraints do not change. Conditions that ensure these assumptions are beyond the scope of this paper.

**IV. Experimental Results**

The biped machine control system has been implemented with an IBM PC/AT and thirteen Intel 8097 microcontrollers.

Data communication between the PC and the 8097 controllers was implemented by thirteen RS-232 serial channels. Serial transfer was chosen because it simplifies cabling and is supported by a wide range of commercial products. Although serial communication does not offer high speed, it is satisfactory for system operation at 100 Hz sampling rate with 9600 baud. Serial communication driver routines were written in assembly language. The joint controller is based on a modular design which, because of the I/O features of the 8097, was easily implemented. The joint controller communicates with the host computer and carries out the low level distributed control function. Within the 10 ms sample, the controller performs the required calculations and sends an analog velocity signal to the dc motor driver. Except for the communication driver, other high level software for the PC—off-line trajectory planning, coordinated control and data communication—was programmed in C. The joint controller software was developed in PL/M-96 on the PC and then downloaded to the controllers.

In the experiment we consider the case where both feet are on the ground and the body sways to the right, to the left, and then returns to its original position. Because the motion is constrained on the frontal plane only four joints \( \theta_6^{(1)}, \theta_4^{(1)}, \theta_2^{(1)}, \theta_1^{(1)} \) and \( \theta_6^{(2)}, \theta_4^{(2)}, \theta_2^{(2)}, \theta_1^{(2)} \) are actuated and the other eight joints are locked by axis brakes. Fig. 3 shows the simplified biped model from the frontal plane. Let the state \( x \) and the control \( u \) be represented by

\[ x = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}, \]
\[ u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}. \]

The constraint relation is

\[ \begin{bmatrix} 1 \cos(z_1) \\ 1 \sin(z_1) \end{bmatrix} - \begin{bmatrix} s + 1 \cos(z_4) + s \sin(z_3 + z_4) \\ 1 \sin(z_1) - s \cos(z_4) - s \cos(z_3 + z_4) \end{bmatrix} = 0 \]

\[ \begin{bmatrix} z_1 \\ z_3 + z_4 - z_2 \end{bmatrix} = 0 \]

where

\[ 1 = d_1 + d_2 \]
\[ s = 2w_0 \]

and \( d_1 \) is the length of the upper leg; \( d_2 \) is the length of the lower leg; and \( w_0 \) is half of the width of the body. Let the \( z_1 \)
Let the performance index (18) be specified by

\[ Q = \begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix} \]

\[ R = 1 \]

where \( \rho > 0 \) is a weighting factor. Then the optimal state feedback is

\[ u_1 = \begin{bmatrix} -P_{12} - P_{22} \end{bmatrix} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_1 \end{bmatrix} = [-P_{12} - P_{22}] \begin{bmatrix} z_1 - z_1 \\ z_1 - z_1 \end{bmatrix} \]

(29)

where \( P_{12} = a(\sqrt{\rho + b^2} - b) \) and \( P_{22} = a\sqrt{\rho + 2P_{12}} \). In this case the overall control becomes

\[ u = \dot{u} + \ddot{u} = \begin{bmatrix} (H\ddot{z}_1 + \dot{y})/4 - P_{12}\dot{z}_1 - P_{22}\dot{z}_1 \\ -(H\ddot{z}_1 + \dot{y})/4 + \dot{z}_1 \\ -(H\ddot{z}_1 + \dot{y})/4 + \dot{z}_1 \\ (H\ddot{z}_1 + \dot{y})/4 + \dot{z}_1 \end{bmatrix} \]

(30)

As an example, the desired trajectory for \( \dot{z}_1 \) is shown in Fig. 4(a), and constraints for other joints are as follows:

\[ \dot{z}_2 = -\dot{z}_1, \dot{z}_3 = -\dot{z}_1, \dot{z}_4 = \dot{z}_1. \]

For the comparison, both the PD and proposed control algorithms are used in the same experiment. Figs. 4(b) and 4(c) show the experimental results using the traditional PD control and the proposed algorithm with \( \rho = 2 \), respectively. Fig. 4(d) shows the vertical reaction forces of leg 1 and leg 2 using the PD control. The reaction force may change suddenly if there is a constraint error during the motion due to the position error. Fig. 4(e) shows the vertical reaction forces of leg 1 and leg 2 using the proposed control strategy. The reaction force trajectories are smoother and closer to the ideal trajectories.

V. CONCLUSION

A dynamic model of a biped robot has been formulated in terms of reduced independent variables. A control law has been implemented by feedforward compensation and optimal linear state feedback. Experimental implementation and results of biped sway motion show that its performance is much better than that obtained by conventional PD control. Theoretically, the proposed approach is applicable to the 3-D case. The major difficulty in applying the result to 3-D motion is generation of the dynamic equations of the constrained motion for which computational method has been developed [10]. Extensions of this research include

1) determination of torque distribution in the double-support phase,
2) stability analysis of the system when subjected to feedforward and feedback control,
3) a method to achieve smooth transition between the single-support and double-support phases; and
4) 3-D walking motion.
Fig. 4. Experimental results: (a) Desired joint trajectory for $z_1$; (b) Actual joint trajectories using the PD control; (c) Actual joint trajectories using the proposed control algorithm; (d) Actual vertical ground reaction force using the PD control; and (e) Actual vertical ground reaction force using the proposed control algorithm.

REFERENCES


Ching Long Shih (S'85–M'89) received the B.S. and M.S. degrees from National Chiao Tung University, Taiwan, in 1980, and 1984, respectively and the Ph.D. degree in electrical engineering in 1988 from The Ohio State University, Columbus. Currently, he is an Associate Professor at the Department of Electrical Engineering at the National Taiwan Institute of Technology. His research involves control systems design and implementation for industrial and legged robots, and motion planning of robotics systems.


He is a Professor of Engineering Science at Simon Fraser University, Burnaby, BC, Canada, where he is responsible for research and teaching in robotics and automation. From 1967 to 1991, as Director of the Center for Robotics and Manufacturing Systems at the University of Kentucky, he built the organization from its inception and initiated major research and industrial extension programs. From 1979 to 1988 he held technical management positions in manufacturing automation and product development. At GE's Industrial Electronics Laboratory, Charlottesville, VA, he led the development of robot vision and simulation software. As Manager of the GE Automation Center in Frankfurt, Germany, he established a major technology center for computer integrated manufacturing serving six European countries. As division president at IRT Corporation in San Diego, CA, he managed development of a new product for automated inspection of electronic circuit boards. As Vice President and co-founder of LTI Robotic Systems, in Torrance, CA, he directed R&D and consulting projects in the U.S. and Japan involving automated systems for manufacturing, robot controllers, and robot languages. From 1965 to 1979 he held positions at NASA's Marshall Space Flight Center, DFVLR German Space Research Establishment, Technische Hochschule Darmstadt, United States Naval Academy, and North Carolina State University. In 1973 he received the Humboldt Senior Scientist Award for international research contributions.

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